

How to Make a Decision Scientifically: The Analytic Hierarchy Process

Introduction

We are all fundamentally decision makers. Everything we do consciously or unconsciously is the result of some decision. The information we gather is to help us understand occurrences in order to develop good judgments to make decisions about these occurrences. Not all information is useful for improving our understanding and judgments. If we only make decisions intuitively, we are inclined to believe that all kinds of information are useful and the larger the quantity the better. But that is not true. There are numerous examples which show that too much information is as bad as little information. Knowing more does not guarantee that we understand better as illustrated by some author's writing "Expert after expert missed the revolutionary significance of what Darwin had collected. Darwin, who knew less, somehow understood more." To make a decision we need to know the problem, the need and purpose of the decision, the criteria of the decision, their sub-criteria, stakeholders and groups affected and the alternative actions to take. We then try to determine the best alternative, or in the case of resource allocation we need priorities for the alternatives to allocate their appropriate share of the resources.

Decision making, for which we gather most of our information, has become a mathematical science today (Figuera et al.2005). It formalizes the thinking we use so that what we have to do to make better decisions is transparent in all its aspects. We need to have some fundamental understanding of this most valuable process that nature endowed us with to make it possible for us to make choices that help us survive. Decision making involves many criteria and sub-criteria used to rank the alternatives of a decision. Not only does one need to create priorities for the alternatives with respect to the criteria or sub-criteria in terms of which they need to be evaluated, but also for the criteria in terms of a higher goal, or if they depend on the alternatives, then in terms of the alternatives themselves. The criteria may be intangible and have no measurements to serve as a guide to rank the alternatives and creating priorities for the criteria themselves in order to weight the priorities of the alternatives and add over all the criteria to obtain the desired overall ranks of the alternatives is a challenging task. How? In the limited space we have, we can only cover some of the essentials of multi-criteria decision making, leaving it to the reader to learn more about it from the literature cited at the end of this chapter.

The measurement of intangible factors in decisions has for a long time defied human understanding. Number and measurement are the core of mathematics and mathematics is essential to science. So far mathematics has assumed that all things can be assigned numbers from minus infinity to plus infinity in some way and all mathematical modeling of reality has been described in this way by using axes and geometry. Naturally all this is predicated on the assumption that one has the essential factors and all these factors are measurable. But there are many more important factors that we do not know how to measure than there are ones that we have measurements for. Knowing how to measure such factors could conceivably lead to new and important theories that rely on



many more factors for their explanations. After all, in an interdependent universe everything depends on everything else. Is this just a platitude or is there some truth behind it? If we knew how to measure intangibles, much wider room would be open to interpret everything in terms of many more factors than we have been able to do so far scientifically. One thing is clear, numerical measurement must be interpreted for meaning and usefulness according to its priority to serve our values in a particular decision. It does not have the same priority for all problems. Its importance is relative. Therefore, we need to learn about how to derive relative priorities in decision making.

Background

There are two possible ways to learn about anything - an object, a feeling or an idea. The first is to examine and study it in itself to the extent that it has various properties, synthesize the findings and draw conclusions from such observations about it. The second is to study that entity relative to other similar entities and relate it to them by making comparisons.

The cognitive psychologist Blumenthal (1977) wrote that "Absolute judgment is the identification of the magnitude of some simple stimulus...whereas comparative judgment is the identification of some relation between two stimuli both present to the observer. Absolute judgment involves the relation between a single stimulus and some information held in short-term memory, information about some former comparison stimuli or about some previously experienced measurement scale... To make the judgment, a person must compare an immediate impression with impression in memory of similar stimuli"

Using judgments has been considered to be a questionable practice when objectivity is the norm. But a little reflection shows that even when numbers are obtained from a standard scale and they are considered objective, their interpretation is always, I repeat, always, subjective. We need to validate the idea that we can use judgments to derive tangible values to provide greater credence for using judgments when intangibles are involved.

The Analytic Hierarchy Process

To make a decision in an organized way to generate priorities we need decompose the decision into the following steps.

1. Define the problem and determine the kind of knowledge sought.

2. Structure the decision hierarchy from the top with the goal of the decision, then the objectives from a broad perspective, through the intermediate levels (criteria on which subsequent elements depend) to the lowest level (which usually is a set of the alternatives).

3. Construct a set of pairwise comparison matrices. Each element in an upper level is used to compare the elements in the level immediately below with respect to it.



4. Use the priorities obtained from the comparisons to weight the priorities in the level immediately below. Do this for every element. Then for each element in the level below add its weighted values and obtain its overall or global priority. Continue this process of weighting and adding until the final priorities of the alternatives in the bottom most level are obtained.

To make comparisons, we need a scale of numbers that indicates how many times more important or dominant one element is over another element with respect to the criterion or property with respect to which they are compared. Table 1 exhibits the scale. Table 2 exhibits an example in which the scale is used to compare the relative consumption of drinks in the United States. One compares a drink indicated on the left with another indicated at the top and answers the question: How many times more, or how strongly more is that drink consumed in the US than the one at the top? One then enters the number from the scale that is appropriate for the judgment: for example enter 9 in the (coffee, wine) position meaning that coffee consumption is 9 times wine consumption. It is automatic that 1/9 is what one needs to use in the (water, coffee) position, and ½ in the (coffee, water) position. One always enters the whole number in its appropriate position and automatically enters its reciprocal in the transpose position.

The priorities, (obtained in exact form by raising the matrix to large powers and summing each row and dividing each by the total sum of all the rows, or approximately by adding each row of the matrix and dividing by their total) are shown at the bottom of the table along with the true values expressed in relative form by dividing the consumption of each drink (volume) by the sum of the consumption of all drinks. The information about actual consumption was obtained from the US Statistical Abstracts. We see the answers are very close and pair-wise comparison judgments of someone who knows can lead to very accurate results of drink consumption.

Intensity of		
	Definition	Explanation
Importance		
1	Equal Importance	Two activities contribute equally to the objective
2	Weak or slight	
3	Moderate importance	Experience and judgment slightly favor one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgment strongly favor one activity over another
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice
8	Very, very strong	•
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation
D : 1	If activity <i>i</i> has one of the above nonzero numbers	
Reciprocals	assigned to it when compared	
	with activity <i>i</i> , then <i>i</i> has the	A reasonable assumption
of above	reciprocal value when	
	compared with <i>i</i>	

 Table 1
 The Fundamental Scale of absolute numbers



 Table 2 Relative consumption of drinks

Which Drink is Consumed More in the U.S.?

Drink	An	Exan	nple	of Est	imatio	n Usin	g Jud	gment
Consumpt in the U.S.	ion Co	offee	Wine	Tea	Beer	Sodas	Milk	Water
Coffee	(1		9	5	2	1	1	1/2
Wine	1/9	9	1	1/3	1/9	1/9	1/9	1/9
Tea	1/:	5	2	1	1/3	1/4	1/3	1/9
Beer	1/2	2	9	3	1	1/2	1	1/3
Sodas	1		9	4	2	1	2	1/2
Milk	1		9	3	1	1/2	1	1/3
Water	2		9	9	3	2	3	1
	The de	rived s	cale ba	sed on tl	he judgm	ents in th	e matri	x is:
	Coffee .177	Wine .019	Tea .042	Bee 2	er Sod 6 .190	as Milk) .129	: Wa .32	ter 7
	with a	consist	ency ra	atio of .0	22.			
	The ac	tual con	nsump	tion (from	n statisti	cal source	es) is:	
	.180	.010	.04	.12	0.180	.140	.33	0

An Example of a Simple Decision

The following is a simple decision examined by someone to determine what kind of job would be best for her after getting her PhD: either to work at two kinds of companies or to teach at two kinds of schools. The Goal is to determine the kind of job for which she is best suited as spelled out by the criteria. Because of space limitations we will not define them in detail here. For more detail see Saaty, (1994 and 2000).



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Figure 1 Best job decision

There are 12 pairwise comparison matrices in all: One for the criteria with respect to the goal, which we show here in Table 3, two for the sub-criteria the first of which for the sub-criteria under flexibility: location, time, and work, that we show in Table 4 and one for the sub-criteria under opportunity that we do not show here. Then there are nine comparison matrices for the four alternatives with respect to all the "covering criteria", the lowest level criteria or sub-criteria connected to the alternatives. The 9 covering criteria are: flexibility of location, time and work, entrepreneurial company, possibility for salary increases and a top-level position, job security, reputation and salary. The first six are sub-criteria in the second level and the last three are criteria from the first level. We only show one of these 9 matrices comparing the alternatives with respect to potential increase in salary in Table 5.

In Table 1 the criteria listed on the left are one by one compared with each criterion listed on top as to which one is more important with respect to the goal of selecting a best job. In Table 2 the sub-criteria on the left are compared with the sub-criteria on top as to their importance with respect to flexibility. In Table 3 the alternatives on the left are compared with those on top with respect to relative preference for potential increase in salary. The sub-criteria priorities in Table 2 are weighted by the priority of their parent criterion flexibility (.036) to obtain their global priority.



Table 3 Pairwise comparison matrix of the main criteria with respect to the Goal

	Flexibility	Opportunities	Security	Reputation	Salary	Priorities
Flexibility	1	1/4	1/6	1/4	1/8	.036
Opportunities	4	1	1/3	3	1/7	.122
Security	6	3	1	4	1/2	.262
Reputation	4	1/3	1/4	1	1/7	.075
Salary	8	7	2	7	1	.506

Table 4 Pairwise comparison matrix for the sub-criteria with respect to Flexibility

	Location	Time	Work	Priorities
Location	1	1/3	1/6	.091
Time	3	1	1/4	.218
Work	6	4	1	.691

Table 5 Pairwise comparison matrix for the alternatives with respect to Potential increase in salary

	Domestic Co	Int'l Co	College	State Univ.	Priorities
Domestic company	1	4	3	6	.555
Int'l company	1/4	1	3	5	.258
College	1/3	1/3	1	2	.124
State University	1/6	1/5	1/2	1	.064

The priorities for each matrix are obtained as they were from the matrix of comparisons for the drinks in the US. In Table 6 the rankings of the alternatives are shown against the nine covering criteria (only one of the matrices leading to the rankings was shown, in Table 5). We need to multiply each ranking by the priority of its criterion or sub-criterion and add the resulting weights for each alternative to get its final priority. We call this part of the process, synthesis. It is shown in Table 6. Because Table 6 is horizontally long, it is divided into two pieces where the lower piece follows to the right of the upper piece.

Table 6 Synthesizing to obtain the final results

Criteria	Flexibility			Future Opportunity			
	0.036			0.122			
Subcriteria	Location	Time	Work	Entrepren-	Salary	Top Level	
				eurial	Increases	Position	
	0.091	0.218	0.691				
				0.105	0.637	0.258	
Global weights							
(criteria x subcriteria)							
	0.003	0.008	0.025	0.013	0.078	0.032	
Domestic Company	0.295	0.084	0.062	0.090	0.555	0.591	
Internatn'l Company	0.496	0.055	0.115	0.061	0.258	0.274	
College	0.131	0.285	0.249	0.239	0.124	0.083	
State University	0.078	0.576	0.574	0.610	0.064	0.052	



Criteria	Security	Reputation	Salary	
	0.262	0.075	0.506	
Subcriteria				
Global weights				Overall
(criteria x subcriteria)	0.262	0.075	0.506	Priority
Domestic Company	0.225	0.064	0.124	0.193
Internatn'l Company	0.054	0.101	0.547	0.333
College	0.095	0.247	0.289	0.214
State University	0.626	0.588	0.039	0.262

The overall priorities for the alternative jobs, shown on the far right of the lower piece of Table 6, are the sums across each row for the alternatives. Note that they sum to 1. These priorities may also be expressed in the ideal form by dividing each priority by the largest one, .333 for *International Company*, as shown in Table 7. The effect is to make this alternative the ideal one with the others getting their proportionate value. One may then interpret the results to mean that a *State University* job is about 78% as good as one with an *International Company* and so on.

Name	Normalized	Idealized
DomesticCompany	0.193	0.579
Internatn'l Company	0.333	1.000
College	0.214	0.643

0.262

0.785

State University

Table 7 Final results shown as normalized priorities and idealized priorities

The Ratings Mode

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There is another method to obtain priorities for the alternatives. Here we establish rating categories for each covering criterion and prioritize the categories by pair-wise comparing them for preference. Alternatives are evaluated by selecting the appropriate rating category on each criterion.

The rating categories for the Job Security criterion are Hi, Medium and Low. We compare them for preference using a pair-wise comparison matrix in the usual way as shown in Table 8 below. To obtain the idealized priorities normalize by dividing by the largest of the priorities. The idealized priorities are always used for ratings.



Table 8 Deriving Priorities for Ratings on Job Security

Job Security	Hi	Medium	Low	Priorities	Idealized Priorities
Hi	1	3	7	0.6586	1.0000
Medium	1/3	1	4	0.2628	0.3989
Low	1/7	1/4	1	0.0786	0.1193

The rating categories for all the covering criteria and their priorities are established in a similar way and are shown in Table 9.

Job Security Hi	Reputation Excellent	Salary >3000	Location Flexibility Very Flexible	Time Flexibility Flexible	Work Flexibility Very Flexible	Entre- prenual Very Probable	Salary Increase High	Top Level Likely
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Med	Above Avg	2000-3000	Flexible	Avg 0.405	Flexible	Probable	Above Avg	Depends
0.399	0.487	0.551	0.431		0.665	0.373	0.299	0.399
Low	Average	1500-2000	Avg Flexible	Not Flexible	Avg Flexible	Not Probable	Average	Not Likely
0.119	0.403	0.217	0.187	0.164	0.423	0.113	0.119	0.119
	Below Avg	1000-1500	Not flexible		Not flexible	Impossible		
	0.171	0.162	0.124		0.195	0.075		
	Poor	<1500	Stable		Stable			
	0.113	0.059	0.087		0.122			

Table 10 Ratings for the Alternatives on each Criterion

	Totals	Priorities	3JobSecurity 0.261899	4Reputation 0.074559	5Salary 0.505585	LocationFlexibility 0.003296
International Compa	0.619340	0.310067	Low	Above Average	>3000	Very Flexible
Domestic Company	0.363962	0.182214	Medium	Average	1500-2000	Flexible
State University	0.505153	0.252900	High	Excellent	1000-1500	Stable
College	0.508984	0.254818	Medium	Excellent	2000-3000	Not Flexible

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TimeFlexibility 0.007846	WorkFlexibility 0.024909	Entreprenual 0.012766	SalaryIncrease 0.077653	TopLevel 0.031487
Not Flexible	Not Flexible	Impossible	Above Avg.	Depends
Avg.	Not Flexible	Impossible	High	Likely
Flexible	Very Flexible	Very probable	Avg.	Likely
Flexible	Flexible	Probable	Avg.	Depends

Table 11 Numerical values for Ratings shown in Table 10

	Criterion				
	Priorities	0.262	0.075	0.506	0.003
		Job	Reputation	Salary	Location
TOTALS	PRIORITIES	Security			Flexibility



International Company	0.619	0.310	0.119	0.487	1.000	1.000	
Domestic Company	0.364	0.182	0.399	0.403	0.217	0.431	
State University	0.505	0.253	1.000	1.000	0.162	0.087	
College	0.509	0.255	0.399	1.000	0.551	0.124	
			0.008	0.02	.01	0.078	0.031
			Time Flexibility	Work Flexibility	Entre prenual	Salary Increase	Top Level
			0.164	0.195	0.075	0.299	0.399
	•	••	0.405	0.195	0.075	1.000	1.000
			1.000	1.000	1.000	0.119	1.000
			1.000	0.665	0.373	0.119	0.399

Table 10 shows the verbal ratings of the four alternatives on each covering criterion and Table 11 shows their corresponding numerical ratings from Table 9 with their totals shown in the first column on the left. The totals are converted to priorities by dividing by their sum in the second column on the left.

Comparing the results from the pair-wise comparison method called a relative model to these results from the ratings model as shown in Table 12 we note that the first two alternatives' priorities are very close. The last two are a little different. This is to be expected. The two methods do not deliver the same priorities exactly. The relative model method where alternatives are compared with each other under the various criteria is more accurate. The ratings method has the advantage that one can rate large numbers of alternatives rather quickly, and the results are adequately close.

	Relative	Ratings
	Model	Model
	Results	Results
DomesticCompany	0.192	0.182
Int.Company	0.333	0.310
College	0.214	0.255
StateUniv.	0.261	0.253

 Table 12 Comparing Relative and Rating Results

The process of paired comparisons has far broader uses for making decisions. We can deal with a decision from four different standpoints: The benefits (B), that the decision brings, the opportunities (O) it creates, the costs (C) that it incurs and the risks (R) that it might have to face. We refer to these merits together as BOCR. Some people in the field of strategic planning use similar factors known as SWOT (strengths, weaknesses, opportunities and threats) having switched the order of weaknesses and opportunities in making the correspondence with BOCR. The alternatives must be ranked for each of the four merits. The four ranking are then combined into a single overall ranking by rating the best alternative in each of the BOCR on strategic criteria that an individual or a government uses to decide whether or not to implement one or the other of



the numerous decisions that they face. The results of the four ratings determine the priorities each of which is used to weight all the priorities of all the alternatives with respect to that merit.

There is in addition the possibility of the dependence of the criteria on the alternatives in addition to the mandatory dependence of the alternatives on the criteria or among themselves. In that case we have a decision with dependence and feedback. To determine the best course of action in such decisions needs a few days to do thoroughly. Its steps are carried out along the lines indicated in Figure 1.



Figure 1 Steps followed in the Analytic Network decision process

Group Decision Making

Two important issues in group decision making are: how to aggregate individual judgments in a group into a single representative judgment for the entire group and how to construct a group choice from individual choices. The reciprocal property plays an important role in combining the judgments of several individuals to obtain a single judgment for the group. Judgments must be combined so that the reciprocal of the synthesized judgments is equal to the syntheses of the reciprocals of these judgments. It has been proved that the geometric mean, not the frequently used arithmetic mean, is the only way to do that. If the individuals are experts, they may not wish to combine their judgments but only their final outcomes obtained by each from their own hierarchy. In that case one takes the geometric mean of the final outcomes. If the individuals have different priorities of importance, their judgments (final outcomes) are raised to the power of their priorities and then the geometric mean is formed.



Future Trends

There are two areas that need greater attention in decision making. One is the integration and cataloguing of the structure of a variety of carefully studied decisions to create a dictionary to serve as a source of reference for others to consult so they can benefit from the knowledge that went into making these decisions. Two successful attempts have already been made in this direction resulting in two books: The Hierarchon (Saaty and Forman, 1993) a dictionary of hierarchically structured decisions and the Encyclicon (Saaty and Ozdemir 2005), a dictionary of more general network structured decisions.

Another important area of investigation is how to factor psychological time into a decision in order to anticipate and deal with the future more successfully through prediction and planning. Many efforts are under way in this direction. Books and articles have been published that deal with the future and with planning using the prioritization process described in this chapter.

Conclusion

It appears inescapable that we need an organized way to make decisions and collect information relevant to them when a group must decide by laying out all the important factors and negotiating their understanding, beliefs and values. Here are a few examples where the process has been used in practice.

The Analytic Hierarchy Process has been used in various settings to make decisions.

- In (2001) it was used to determine the best relocation site for the earthquake devastated Turkish city Adapazari.
- British Airways used it in 1998 to choose the entertainment system vendor for its entire fleet of airplanes
- A company used it in 1987 to choose the best type of platform to build to drill for oil in the North Atlantic. A platform costs around 3 billion dollars to build, but the demolition cost was an even more significant factor in the decision.
- The process was applied to the U.S. versus China conflict in the intellectual property rights battle of 1995 over Chinese individuals copying music, video, and software tapes and CD's. An AHP analysis involving three hierarchies for benefits, costs, and risks showed that it was much better for the U.S. not to sanction China. Shortly after the study was complete, the U.S. awarded China most-favored nation trading status and did not sanction it.
- Xerox Corporation has used the AHP to allocate close to a billion dollars to its research projects.



- In 1999, the Ford Motor Company used the AHP to establish priorities for criteria that improve customer satisfaction. Ford gave Expert Choice Inc, an Award for Excellence for helping them achieve greater success with its clients.
- In 1986 the Institute of Strategic Studies in Pretoria, a government-backed organization, used the AHP to analyze the conflict in South Africa and recommended actions ranging from the release of Nelson Mandela to the removal of apartheid and the granting of full citizenship and equal rights to the black majority. All of these recommended actions were quickly implemented.
- The AHP has been used in student admissions, military personnel promotions, and hiring decisions.
- In sports it was used in 1995 to predict which football team would go to the Superbowl and win (correct outcome, Dallas won over my hometown, Pittsburgh). The AHP was applied in baseball to analyze which Padres players should be retained.
- IBM used the process in 1991 in designing its successful mid-range AS 400 computer. IBM won the prestigious Malcolm Baldrige award for Excellence for that effort. Bauer et al. (1992) devoted a chapter on how AHP was used in benchmarking.
- Several military and political applications have been made. Of general interest was the analysis of the decision as to whether to build or not build the national missile defense (NMD) made two years prior to the time that decision was made in December 2002. The decision was the same as the study recommended: build it.

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Key Terms and Their Definitions

Alternative: The possible outcome of a decision. It can be a physical object, a strategy or an action.

Benefits: The advantages, gains or positive values obtained in making a decision.

Comparison: Examination for dominance with respect to a common property.

Costs: The disadvantages or negative values incurred in making a decision.

Criterion: An attribute or condition that an alternative must satisfy.

Element: A single source of influence in a decision.

Goal: The object of a decision.

- **Hierarchy:** A multi-level structure used to represent a decision in which the goal of the decision is at the top, followed by a level of criteria and then another level of sub-criteria and finally the alternatives of the decision always at the bottom. Influences in a hierarchy are linear and run from the top down or from the bottom up.
- Ideal: The best of a group of elements being compared.
- **Network:** A structure for representing decisions that unlike a hierarchy does not have an ordering of levels. Influences are non-linear and run from a group of elements to another and back directly or through a cycle that passes through other groups of elements. The group of alternatives must always receive priorities from other groups of elements, but can also be a source of influence in some decision networks.

Opportunities: The potential (future) advantages, gains or positive values that might result from making a decision.

Pair-wise Comparison: A judgment from the fundamental scale that uses the smaller element as the unit and estimates the larger element to have the attribute a multiple of that unit.

Priority: Relative value of importance.

Rank: Position or order in a group



- **Rate:** To rank by estimating the merit or intensity with which a given alternative in a decision possesses a certain property.
- **Risks:** The potential (future) disadvantages, losses or negative values that might result from making a decision.

Strategic Criteria: Criteria used to evaluate the BOCR merits of a decision to derive priorities for the BOCR by rating their top alternatives. These priorities are used to combine the four different rankings one with respect to each of the merits. The costs and risk are subtracted from the benefits and opportunities.

Sub-criterion: A smaller partition of a criterion.