Decision Making in Complex Environments

Lecture 2 – Ratings and Introduction to Analytic Network Process

Lectures Summary

Lecture 1
- AHP=Hierarchies

Lecture 2
- **Ratings=Large number of alternatives**
- **Convert AHP to ANP model**

Lecture 3
- ANP=Networks

Lecture 4
- Benefits Opportunities Costs Risks

Lecture 5
- Structuring Complex Models with BOCR

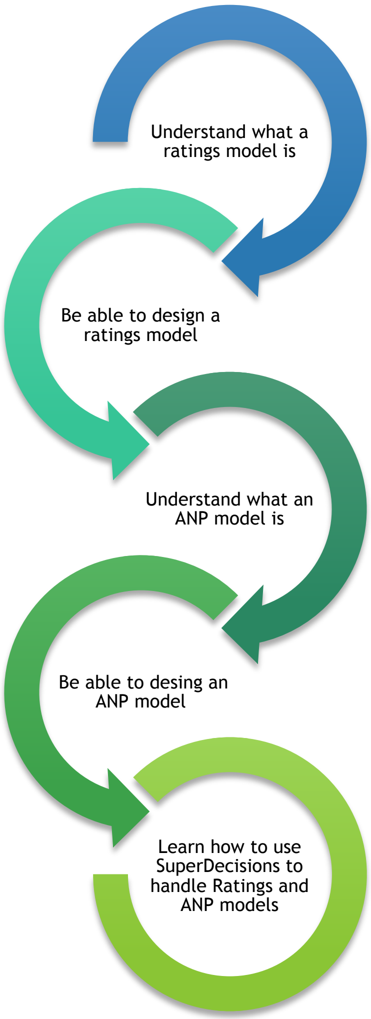
Lecture 6
- Resource Allocation
- Group Decisions

Week 7
- Final Projects

Main points

- Ratings vs Pairwise Comparisons
- Analytic Network Process – ANP
- Use SuperDecisions software to learn how to
  - Create a ratings model
  - Convert an AHP model to ANP model

Outcomes of this lecture



Ratings

What are ratings?

- In AHP/ANP we have two ways of creating priorities:
  - By comparing the alternative in pairs (pairwise comparisons)
  - By RATING the alternatives one at a time with respect to an ideal or standard
    - This kind of measuring is analogous to measuring something with a physical device, like measuring length with a yardstick
- When do we use ratings?
  - When alternatives are though to be independent of one another
    - The presence or absence of an alternative must have no effect on how one rates any of the others
  - When we can have an IDEAL alternative in mind to compare with
  - When the number of alternatives make the pairwise comparisons too time consuming
    - e.g. if we want to evaluate 50 employees, then 1,225 (50(49)/2) pairwise comparisons, would be required for each criterion if we used a pairwise comparison model and not a ratings model.

Ratings - Methodology

Step 1
- Create and AHP or ANP model WITHOUT any alternatives
- No alternatives cluster will appear in the model

Step 2
- Do all the pairwise comparisons needed to evaluate the criteria

Step 3
- Select all the bottom level criteria and use them as criteria in the ratings model

Step 4
- For each criterion define a custom scale

Step 5
- Rate each alternative against the criteria using the respective scale

Step 6
- Calculate each alternative’s score by multiplying the scale value to the criterion weight and sum through the line

Step 7
- Idealize all results to find what is the rank of each alternatives

Ratings and Rank reversal

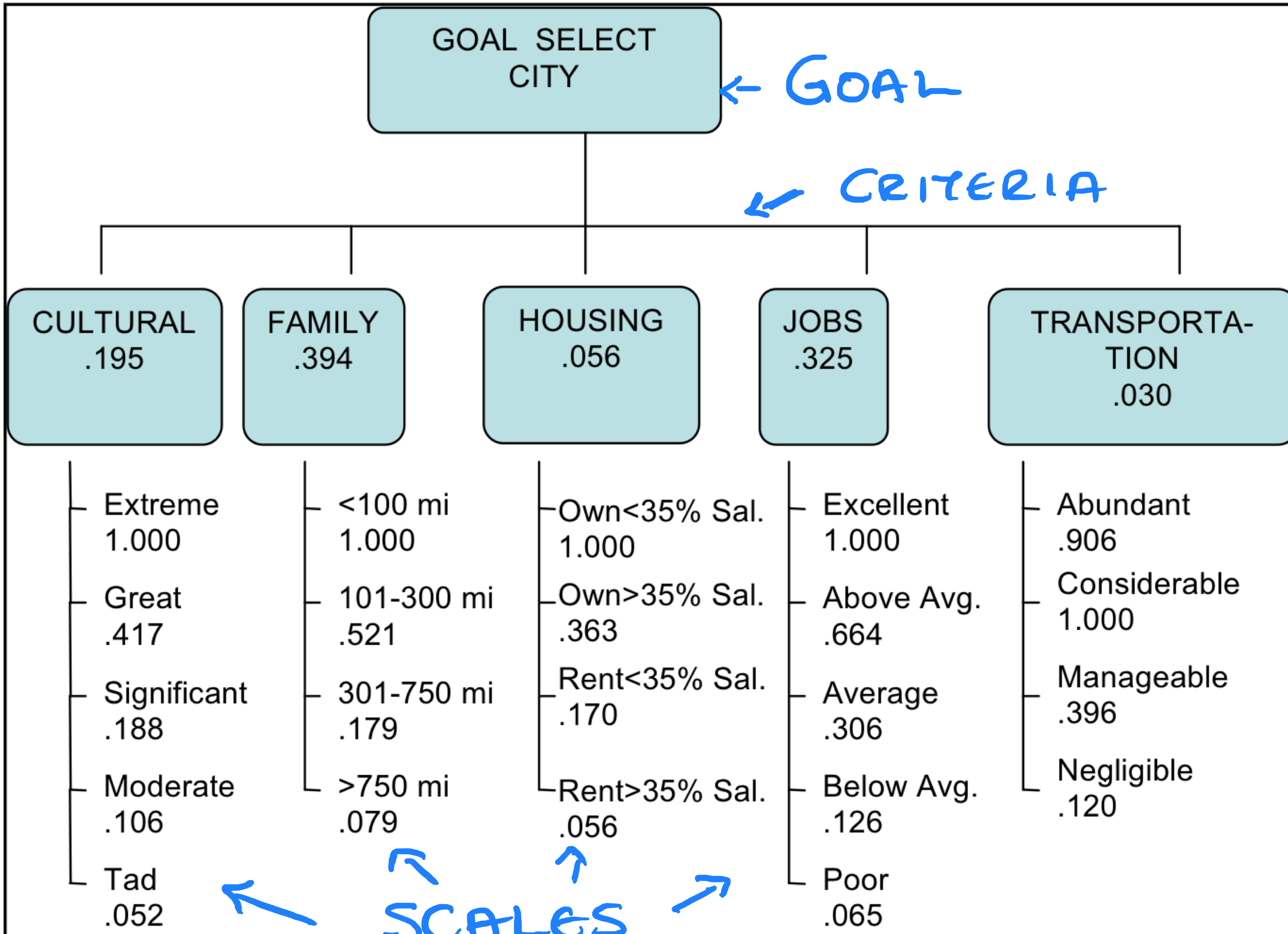
- When using rating we don’t have rank reversals
  - That means that adding alternatives to the model will not create changes on the relative ranks of the existing alternatives
  - In pairwise comparisons this is not the case

Rank reversal

Ratings scales

- One must be able to say how high or low an alternative **rate** on each criterion
- To do that you must have something in mind called an **IDEAL** so that you get the feeling about how close or far the alternative is from the ideal and allocate it to one of various intensity slots of ranking — we call them **SCALES**
- Scales
  - A scale is a group of intensities used to evaluate an alternative against a specific criterion
  - We calculate each scale item's intensity by pairwise comparing the scale items or by directly entering values
  - We have one scale for each decision criterion
  - The same scale can be used for more than one criteria in the same model
  - The same scale can be re-used in different models

Ratings – Example: Best City to Live in



Step 2 – Pairwise compare criteria

- We pairwise compare the criteria to find their weights

	Cultural	Family	Housing	Jobs	Transportation
Cultural	1				
Family		1			
Housing			1		
Jobs				1	
Transportation					1

Step 3 – Create Scale for each criterion

- For each criterion we create a scale, to do so we
  - Find the scale items that are appropriate for that kind of criterion
  - Pairwise compare them to calculate the priorities or
  - Directly enter the values

	Extreme	Great	Significant	Moderate	Tad	*Derived Priorities*	*Idealized Priorities*
Extreme	1	5	6	8	9	*.569*	*1.000*
Great	1/5	1	4	5	7	*.234*	*.411*
Significant	1/6	1/4	1	3	5	*.107*	*.188*
Moderate	1/8	1/5	1/3	1	4	*.060*	*.106*
Tad	1/9	1/7	1/5	1/4	1	*.030*	*.052*

Step 4 – Rate Alternatives

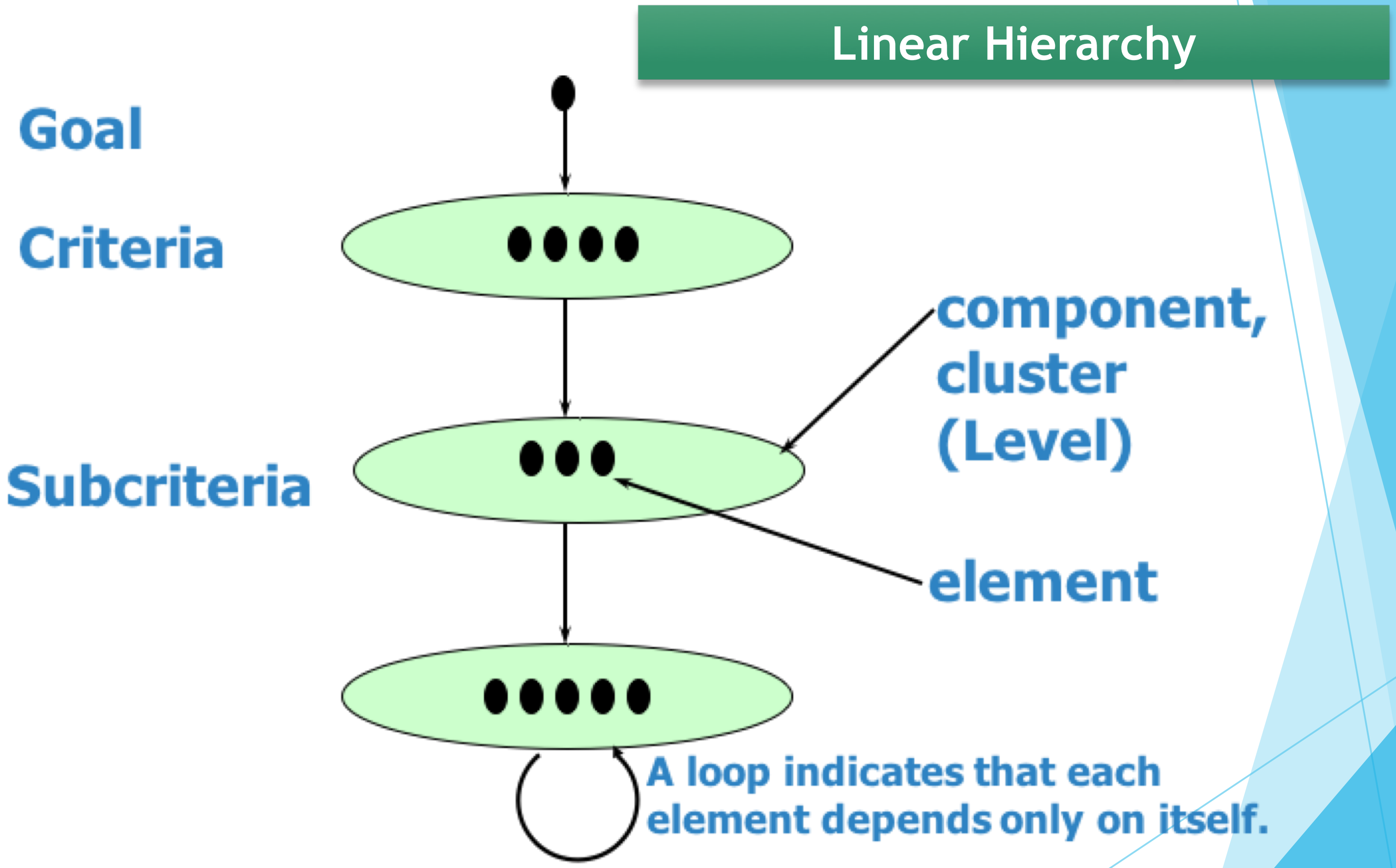
Alternatives	**Cultural** *.195*	**Family** *.394*	**Housing** *.056*	**Jobs** *.325*	**Transport** *.030*	**Total Score**	**Priorities (Normal.)**
Pittsburgh	Signific.	<100 mi	Own>35%	Aver-age	Manage-able	.562	.294
Boston	Extreme	301-750 mi	Rent>35%	Above Avg	Abundant	.512	.267
Bethesda	Great	101-300 mi	Rent<35%	Excel-lent	Consider-able	.650	.339
Santa Fe	Signific.	>750 mi	Own>35%	Aver-age	Negligible	.191	.100

Alternatives	**Cultural** *.195*	**Family** *.394*	**Housing** *.056*	**Jobs** *.325*	**Trans-port** *.030*	**Total Score**	**Priorities (Normalized)**
Pittsburgh	0.188	1.000	0.363	0.306	0.396	.562	.294
Boston	1.000	0.179	0.056	0.664	0.906	.512	.267
Bethesda	0.411	0.521	0.170	1.000	1.000	.650	.339
Santa Fe	0.188	0.079	0.363	0.306	0.120	.191	.100

- Each alternative is rated having in mind how this alternative compares to the ideal alternative
- We calculate the final results by multiplying the value of the cell to the weight of the criterion and sum across the row

Analytic Network Process

AHP

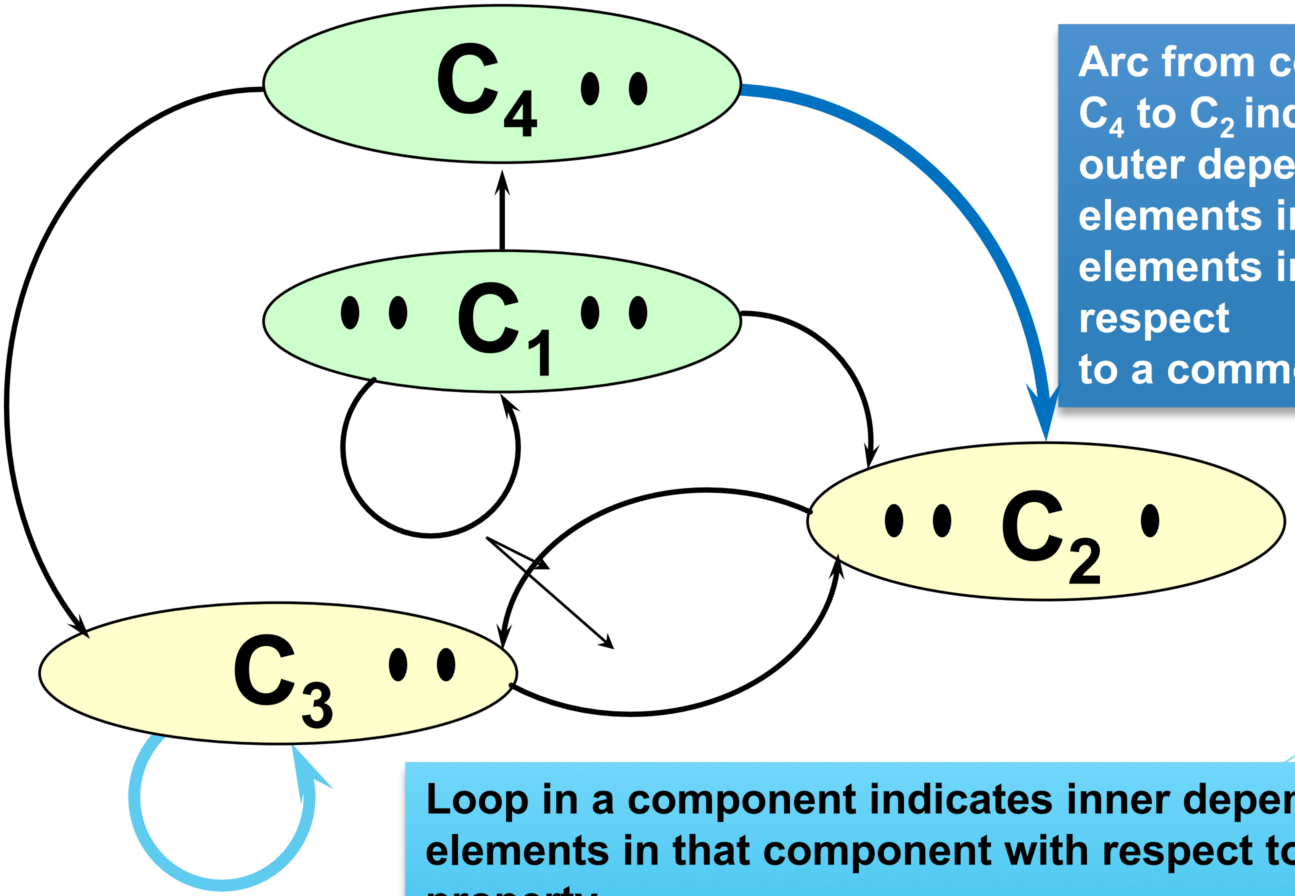

Linear Hierarchy
Goal
Criteria
Subcriteria
component,
cluster
(Level)
element
A loop indicates that each
element depends only on itself.

Analytic Network Process (ANP)

- The ANP is a mathematical theory that makes it possible for one to deal systematically with dependence and feedback, and includes the AHP as a special case.
- The reason for its success is the way it elicits judgments and uses measurement to derive ratio scales.
- Real life problems involve dependence and feedback. Such phenomena can not be dealt with in the framework of a hierarchy but we can by using a network with priorities.
- With **feedback** the alternatives can depend on the criteria as in a hierarchy but <u>may also depend on each other</u>.
- The criteria themselves can <u>depend on the alternatives and on each other</u> as well.
- ***Feedback improves the priorities derived from judgments and makes prediction more accurate.***

Analytic Network Process

Feedback Network with components having
Inner and Outer Dependence among Their Elements



Arc from component C_4 to C_2 indicates the outer dependence of the elements in C_2 on the elements in C_4 with respect to a common property.

Loop in a component indicates inner dependence of the elements in that component with respect to a common property.

AHP vs ANP

- AHP: What is more ***preferred*** or more ***important***? Both are more or less subjective and personal.

- ANP: What has greater ***influence?*** This requires factual observation and knowledge to yield valid answers and thus is more objective.
- Decisions with the ANP should be more stable because one can consider their effect on and survival in the face of other influences.

The questions we answer in ANP

- Given a **criterion**, which of two elements has greater influence (is more dominant) with respect to that criterion?
- Given an **alternative**, which of two criteria or properties is more dominant in that alternative?
- Given a criterion and given an element X in any cluster, which of two elements in the same cluster or in a different cluster has greater influence on X with respect to that criterion?
- The entire decision must use the idea of something "**influencing**" another. Otherwise it must use the idea of "**influenced by**" throughout the analysis as follows:
  - Given a criterion and given an element X in any cluster, which of two elements in the same or in a different cluster is influenced more by X with respect to that criterion.

Main Operations of the ANP

- Relative measurement: Reciprocal relation
- Judgments: Homogeneity
- Hierarchy or Network: Structure of problem; the control hierarchy
- Priorities, Dominance and Consistency: Eigenvector
- Weighting the components
- Composition, Additive to also handle dependence through the supermatrix
- Supermatrix: Interdependence; raising the supermatrix to powers

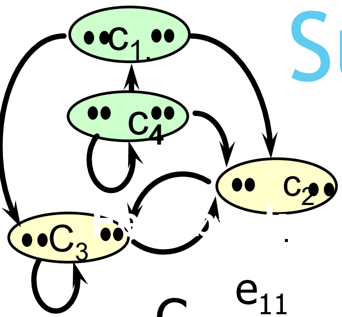
Weighting The Components

- In the ANP one often needs to prioritize the influence of the components themselves on each other component to which the elements belong.
- This influence is assessed through paired comparisons with respect to a control criterion.
- The priority of each component is used to weight the priorities of all the elements in that component.
- The reason for doing this is to enable us to perform feedback multiplication of priorities by other priorities in a cycle, an infinite number of times.
- The process would not converge unless the resulting matrix of priorities is column stochastic (each of its columns adds to one).
- ***To see that one must compare clusters in real life***, we note that if a person is introduced as the president it makes much difference, for example, whether he or she is the President of the United States or the president of a local labor group.

Inner vs. Outer Dependence

- In a network, the elements in a component
  - may influence other elements in the same component (inner dependence)
  - may influence elements in other components (outer dependence)
  - with respect to each of several properties
- We want to determine the overall influence of all the elements. To do so we:
  - organize the properties or criteria
  - define connections among criteria and alternatives
  - perform comparisons
  - synthesize to obtain the priorities of these properties
  - derive the influence of elements in the feedback system
  - weight the resulting influences
  - obtain the overall influence of each element.

Networks and the Supermatrix



$$
\begin{array}{cc}
 & \begin{array}{cccc} C_1 & C_2 & \cdots & C_N \\ e_{11}e_{12}\cdots e_{1n_1} & e_{21}e_{22}\cdots e_{2n_2} & & e_{N1}e_{N2}\cdots e_{Nn_N} \end{array} \\
W = \begin{array}{cc} C_1 & \begin{array}{c} e_{11} \\ e_{12} \\ \vdots \\ e_{1n_1} \end{array} \\ C_2 & \begin{array}{c} e_{21} \\ e_{22} \\ \vdots \\ e_{2n_2} \end{array} \\ \vdots & \\ C_N & \begin{array}{c} e_{N1} \\ e_{N2} \\ \vdots \\ e_{NuN} \end{array} \end{array} &
\begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1N} \\ W_{21} & W_{22} & \cdots & W_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ W_{N1} & W_{N2} & \cdots & W_{NN} \end{bmatrix}
\end{array}
$$

where

$$W_{ij} = \begin{bmatrix} W_{i1}^{(j_1)} & W_{i1}^{(j_2)} & \cdots & W_{i1}^{(jn_j)} \\ W_{i2}^{(j_1)} & W_{i2}^{(j_2)} & \cdots & W_{i2}^{(jn_j)} \\ \vdots & \vdots & & \vdots \\ W_{in_i}^{(j_1)} & W_{in_i}^{(j_2)} & \cdots & W_{in_i}^{(jn_j)} \end{bmatrix}$$

Three Supermatrices in ANP

1) The **original supermatrix** of column eigenvectors obtained from pairwise comparison matrices of elements
2) **Weighted supermatri**x in which each block of column eigenvectors belonging to a component is weighted by the priority of influence of that component. This renders the weighted supermatrix column stochastic.
3) The **limit supermatrix** obtained by raising the weighted supermatrix to large powers.

Supermatrix of a Hierarchy

$$
\begin{array}{cc|cccccc|}
 & & C_1 & C_2 & \cdots & C_{N-2} & C_{N-1} & C_N \\
 & & e_{11} \cdots e_{1n_1} & e_{21} \cdots e_{2n_2} & & e_{(N-2)1} \cdots e_{(N-2)\,n_{N-2}} & e_{(N-1)1} \cdots e_{(N-1)\,n_{N-1}} & e_{N1} \cdots e_{Nn_N} \\
C_1 & \begin{matrix} e_{11} \\ \vdots \\ e_{1n_1} \end{matrix} & 0 & 0 & \cdots & 0 & 0 & 0 \\
C_2 & \begin{matrix} e_{21} \\ \vdots \\ e_{2n_2} \end{matrix} & W_{21} & 0 & \cdots & 0 & 0 & 0 \\
 & & 0 & W_{32} & \cdots & 0 & 0 & 0 \\
\vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & & 0 & 0 & \cdots & W_{n-1,\,n-2} & 0 & 0 \\
C_N & \begin{matrix} e_{N1} \\ \vdots \\ e_{Nn_N} \end{matrix} & 0 & 0 & \cdots & 0 & W_{n,\,n-1} & I
\end{array}
$$

$$W = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 \\ W_{21} & 0 & \cdots & 0 & 0 & 0 \\ 0 & W_{32} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & W_{n-1,\,n-2} & 0 & 0 \\ 0 & 0 & \cdots & 0 & W_{n,\,n-1} & I \end{bmatrix}$$

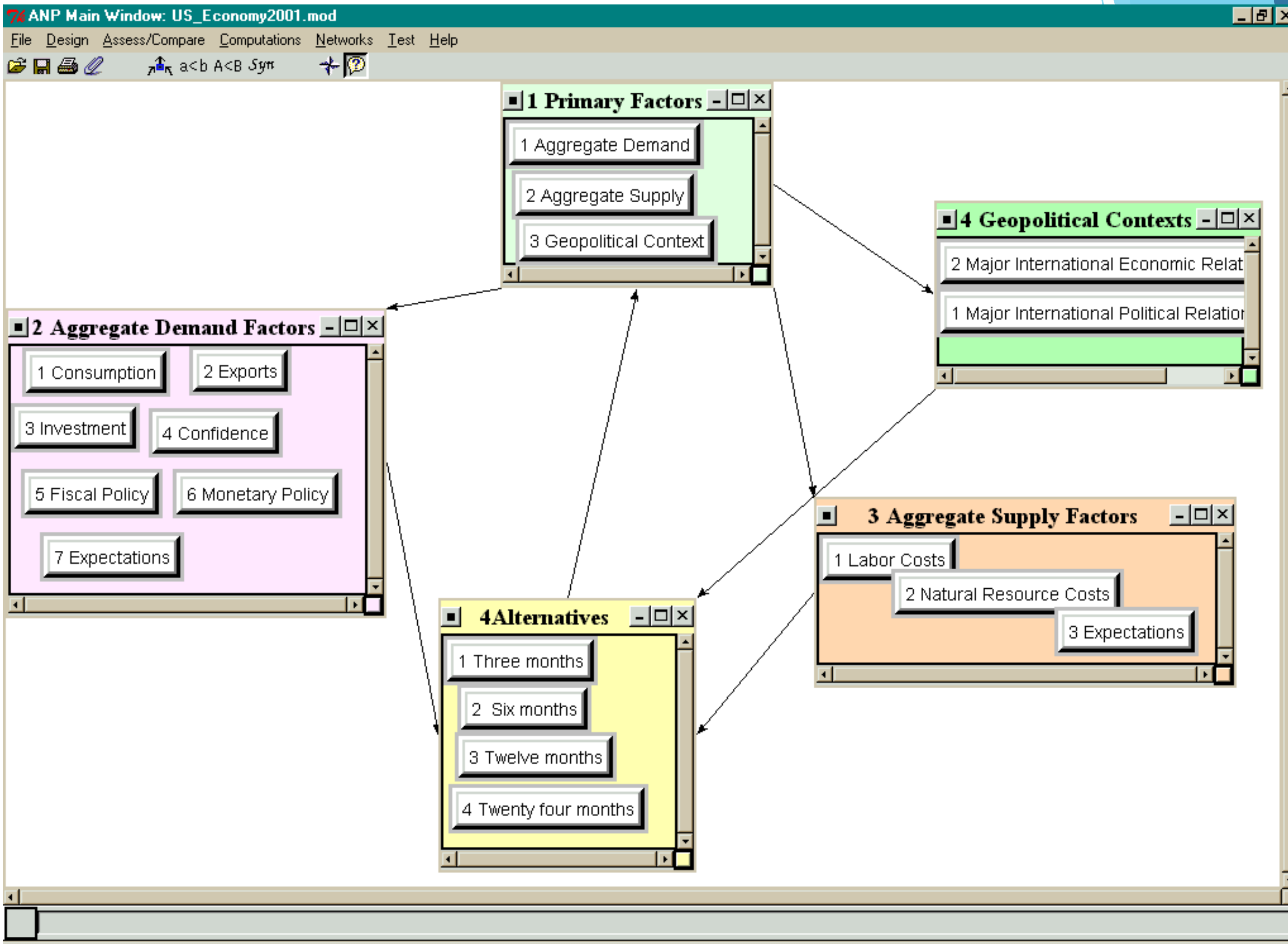
$$
W^k = \begin{bmatrix}
0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 \\
W_{n,n-1} W_{n-1,n-2} \cdots W_{32} W_{21} & W_{n,n-1} W_{n-1,n-2} \cdots W_{32} & \cdots & W_{n,n-1} W_{n-1,n-2} & W_{n,n-1} & I
\end{bmatrix}
$$

for k>n-1

Summarizing ANP

- In ANP we have criteria grouped in clusters and alternatives grouped in a cluster usually named "Alternatives"
- We can have inner and outer dependencies among the criteria, the alternatives and the criteria and the alternatives
- We can have feedback (self loop) in any cluster
- We can pairwise compare the clusters like we do the criteria and the alternatives
- The final results are given by the limit supermatrix
- We do sensitivity in the same way that we do the AHP sensitivity

ANP Example - US Economy Turnaround 2001-2002



ANP Example – Hamburger Model